Int. J. Multiphase Flow Vol. 7, pp. 401–422, 1981
Printed in Great Britain.

0301-9322/81/040401-22$02.00/0
Pergamon/Elsevier

INVESTIGATION OF DROPLET DEPOSITION FROM A TURBULENT GAS STREAM

E. N. GANIĆ and K. MASTANAIAH†

Department of Energy Engineering, University of Illinois at Chicago Circle, Chicago, IL 60680, U.S.A.

(*Received in revised form* 29 *December* 1980)

Abstract—Turbulent deposition of particles from two-phase flow onto the smooth wall of a tube has been studied theoretically and experimentally. A model is proposed for the deposition motion of large particles based on turbulent diffusion in the core followed by a free flight towards the wall. The theory shows that within the Stokes regime, the dimensionless deposition velocity k_d/u^* depends on Re and τ^+ only, where u^* is the friction velocity, Re is the tube Reynolds number and τ^+ is the dimensionless particle relaxation time. Deposition data are obtained for air–water droplet flow through a 12.7-mm i.d. acrylic tubing at Re = 52,500 and 94,600. The proposed theory satisfactorily describes the existing deposition data as well as present measurements, covering a wide range of Re and τ^+.

1. INTRODUCTION

The determination of droplet deposition rates is of interest in many technical applications such as once-through steam generators, nuclear reactor systems, spray cooling and other two-phase flow situations. When a turbulent stream containing suspended particles flows past a wall, particles are deposited on the surface by the action of fluid turbulence, singly or in combination with other mechanisms such as Brownian diffusion, gravitational settling and electrostatic effects. Accurate description of the mechanism of purely turbulence-controlled deposition is essential for the analysis of deposition motion of particles under more complex conditions where other mechanisms are also present simultaneously. A critical examination of the deposition measurements for wide range of particle sizes from submicron to several hundreds of microns has been reported by McCoy & Hanratty (1977). As shown in figure 1 they have presented the dimensionless deposition velocity $k_d^+(=k_d/u^*)$ for vertical systems vs the dimensionless particle relaxation time τ^+ defined as

$$\tau^+ = d^2\rho_G\rho_d u^{*2}/18\mu_G^2 \qquad [1a]$$

based on volume median diameter. Here $k_d = N_0/\bar{c}$, N_0 is the mass rate of deposition of particles per unit area, $\bar{c}$ is the bulk concentration of particles across the tube, u^* is the friction velocity, d and ρ_d are the particle diameter and density, and ρ_G and μ_G are the density and dynamic viscosidy of the fluid respectively. The following observations have been made in view of the data depicted in figure 1.

For particles in the submicron range, $\tau^+ < 0.15$, particles follow the streamlines of fluid motion, and Brownian diffusion is the mechanism responsibile for deposition, suggesting that k_d^+ is independent of τ^+ and is a function of Schmidt number Sc$(=\nu_G/\bar{D})$ only, where ν_G is the kinematic viscosity of the fluid and $\bar{D}$ is the Brownian diffusivity.

When $\tau^+ > 0.15$, k_d^+ is however found to be independent of Brownian motion, and Sc is no longer an important dimensionless group. According to the theory proposed by Friedlander & Johnstone (1957), particles in this range diffuse towards the wall due to radial velocity fluctuations of the turbulent eddies up to one stopping distance from the wall, and then deposit on the wall by a free-flight (inertial) mechanism through the viscous boundary layer, owing to the initial momentum imparted to them by the fluid eddies. A stopping distance, S, is defined as the distance a particle would travel through a stagnant fluid with a prescribed initial velocity under the conditions of Stokesian drag, and is given by $S = (d^2\rho_d v_{p0}/18\mu_G)$, where v_{p0} is the

†Present address: Mechanics and Mechanical and Aerospace Engineering Department, IIT, Chicago, IL 60616, U.S.A.

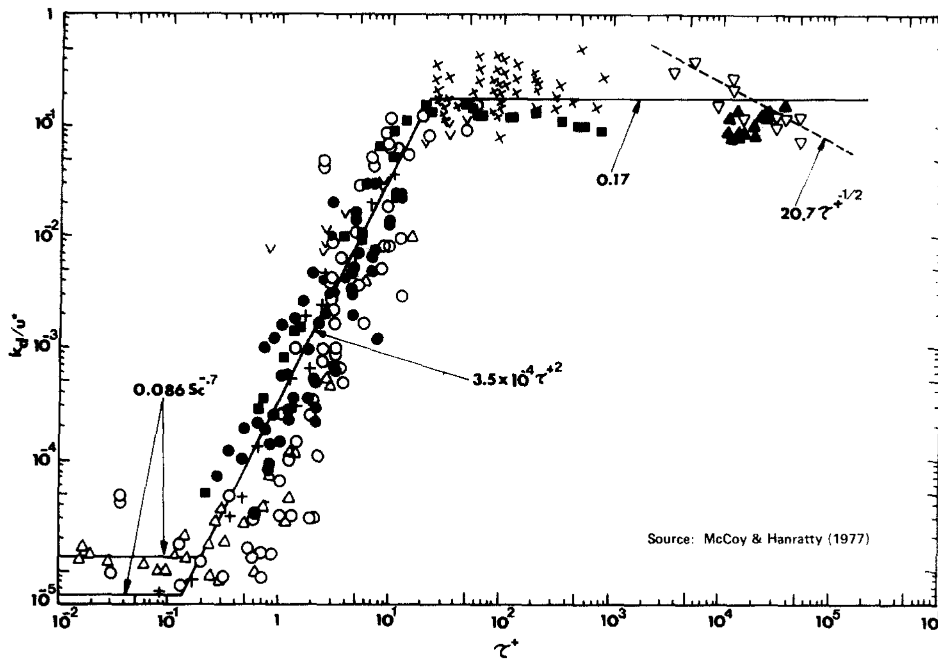

Figure 1. Summary of literature deposition data for vertical flow systems. ▽, Farmer *et al.* (1970); ×, Forney & Spielman (1974); ●, Friedlander & Johnstone (1957); ▲, Cousins & Hewitt (1968); V, Ilori (1971); ■, Liu & Agarwal (1974); +, Schwendimen & Postma (1961); ○, Sehmel (1968); △, Wells & Chamberlain (1967).

initial particle velocity at the start of free flight. The dimensionless stopping distance S^+ is given as $S^+ = Su^*/\nu_G$. The data indicate that k_d^+ is approximately proportional to τ^{+2} up to τ^+ about 20–30.

The experimental deposition data in the above range of τ^+ are reported by Friedlander & Johnstone (1957), Wells & Chamberlain (1967), Sehmel (1968), Liu & Agarwal (1974), Agarwal (1975) and others. In order to better explain the observed deposition data, the theory of Friedlander & Johnstone (1957) was later modified by a number of investigators (Davies 1966, Beal 1968, Sehmel 1970, Liu & Ilori 1973). A detailed survey of these models is given by McCoy (1975).

However as the particle size increases beyond $\tau^+ > 20$–30, none of the above models is able to predict the experimental data. There is observed a marked change in the behavior of k_d^+, suggesting a deviation in the mechanism of particle deposition. Indeed, this range of τ^+ arises in most practical applications involving two-phase dispersed or drop flow. For vertical flow systems, deposition data in this range include those of Sehmel (1968), Ilori (1971), Forney & Spielman (1974), Liu & Agarwal (1974) and Agarwal (1975) for uniform-sized particles, and of Cousins & Hewitt (1968) and Farmer *et al.* (1970) involving a drop size distribution. Unfortunately the accumulated data show a large amount of scatter, thereby masking the separate effects of Re and ρ_d/ρ_G, where Re is the Reynolds number ($= \bar{U}D/\nu_G$), D being the tube diameter and $\bar{U}$, the superficial gas velocity.

A number of theoretical analyses also exist in the literature for predicting the deposition rates in this region of particle size. Friedlander & Johnstone (1957) have suggested that "for $S^+ > 30$, the particles need to diffuse only to the edge of the sublayer" although they have not considered any experimental data to examine the validity of their supposition. It is to be pointed out that for particles with $S^+ > 30$, Friedlander & Johnstone (1957) have assumed equality of particle and fluid diffusivities and employed Reynolds analogy to obtain the deposition velocity that is independent of particle size. This assumption is not realistic for large particles since their response to fluid fluctuations becomes imperfect and uncertain (Owen 1969), and is in contrast with the deposition data of Liu & Agarwal (1974) and Agarwal (1975).

Hutchinson, Hewitt & Dukler (1971) have presented a stochastic model for calculating the deposition flux. The particle motion associated with the eddy interactions in the turbulent core, and in the wall region of highly damped turbulence is considered random. The deposition rate is found to depend on Re, ρ_d/ρ_G, d/D and L/D, where L is the tube length. From the best fit of the theory with the available data, they find that the wall region is described by $0 < y^+ < 1.25$, where $y^+ = yu^*/\nu_G$ and y is the radial distance from the wall. McCoy (1975) points out in the above theory that if the free flights begins at $y^+ = 1.25$, the initial particle velocity should be the one that corresponds to local fluctuating velocity of the fluid, instead of the velocity that exists in the core region. Also Gardner (1975) observes the theory of Hutchinson *et al.* (1971) predicts maximum possible k_d^+ as 0.81, which is about 6 times higher than that measured by Liu & Agarwal (1974).

Forney & Spielman (1974) have presented an expression for the calculation of deposition velocity for particles having $S^+ > 30$. Their theory is a modification of the analysis of Friedlander (1954) which predates the published work of Friedlander & Johnstone (1957). It employs the stopping distance concept and considers that the location where free flight starts can vary from $y^+ = 30$ to $y^+ = r_0^+$, depending on the particle size, where $r_0^+ = r_0u^*/\nu_G$. The boundary condition employed is $c = 0$ at $y^+ = S^+$ for $30 < S^+ < r_0^+$. The main difficulty with the theory of Forney & Spielman (1974) is that the assumption of a zero particle concentration at locations corresponding to the start of free flight in the turbulent core is incorrect, and is in direct contrast with the reported measurements for the concentration distribution (Hagiwara *et al.* 1979). The error in such an assumption becomes increasingly serious when the start of free flight becomes closer to the tube center. Another limitation of their theory is that their analysis fails to describe the deposition process when the stopping distance exceeds the tube radius, in which case turbulent diffusion with free flight as the final transport totally breaks down.

Furthermore, it is widely recognized that the applicability of the stopping distance concept for $S^+ > 30$ is not appropriate (Agarwal 1975). This is evident from the fact that if the free flight were to start at $S^+ > 30$ depending on the particle size, the region $30 < y^+ < S^+$ is not purely viscous, but consists of turbulent fluctuations of high intensity and thereby offers resistance to particle free flight. Therefore particle free flight in the turbulent core becomes questionable.

Cleaver & Yates (1975) have proposed a deposition model based on the observations of the structure of the wall region of a turbulent boundary layer. It is assumed that particles move to a certain height above the surface by turbulent diffusion and are convected to the wall in the downsweep of fluid towards the wall, while some of them are carried back into the turbulent core by turbulent burst (upsweep). However their theory predicts an asymptote as $\tau^+ \rightarrow 1000$, a result that is in contrast with data for large particles (Agarwal 1975). The limitations of the analyses of Namie & Ueda (1973), Reeks & Skyrme (1976), and James & Hutchinson (1979) are also discussed in Mastanaiah (1980).

From the foregoing considerations, it is evident that there exists no satisfactory theory that can explain the mechanism of droplet transport when the particle relaxation time is in excess of about 30. The purpose of the present work is to experimentally and theoretically study the mechanism of turbulence-controlled deposition in the higher range of particle size.

2. ANALYSIS

2.1 *Proposed physical model*

The present model is concerned with particles which have stopping distance S^+ in excess of the combined thickness of the viscous sublayer and the buffer layer ($y^+ = 30$).

The intensity of radial turbulence from the center of the tube ($y^+ = r_0^+$) to the periphery of the turbulent core ($y^+ = 30$) is high and practically uniform as indicated by Laufer's data (Hinze 1975, pp. 725 & 726). Therefore, particles are subjected to the radial velocity fluctuations regardless of S^+ and thereby diffuse to the periphery of the turbulent core. The radial velocity fluctuations outside the turbulent core ($0 < y^+ < 30$) are, however, small compared to those in the core region (Hinze 1975, p. 726) and therefore particle transport towards the wall by turbulent diffusion is questionable. However, all particles with $S^+ > 30$ have already enough momentum, imparted to them by the turbulent eddies at the periphery of the turbulent core, to penetrate the buffer layer and viscous sublayer by inertial flight. The above arguments form the physical basis for our consideration that all particles with $S^+ > 30$ move from the tube center to the edge of buffer layer by turbulent diffusion, and then reach the wall by a free flight mechanism. This is consistent with the notion of Friedlander & Johnstone (1975).

The present work however considers the effect of particle inertia on the deposition rates, and this marks an important difference between the present model and that proposed by Friedlander & Johnstone (1957).

We consider now the case when the particle stopping distance coincides with the edge of the buffer layer, i.e. $S^+ = 30$. It may be noted that the particle relaxation time, τ, is defined as $\tau = S/v_{p0}$, and that the stopping distance, S, and the relaxation time, τ, are made dimensionless with respect to wall parameters u^* and ν_G, resulting in the relationship $\tau^+ = (S^+ u^*/v_{p0})$. At the edge of the buffer layer $y^+ = 30$, the radial rms velocity fluctuation of the fluid, $v_{f0} \simeq 0.75\, u^*$ (Hinze 1975). For the case of $S^+ = 30$, $v_{p0} \simeq v_{f0}$ is a good approximation, as can be justified by the relations to be derived ([8] and [16]) for the particle to fluid diffusivity ratio. Since there is a change in the particle transport mechanism as stated earlier, the relaxation time corresponding to $S^+ = y^+ = 30$ will be termed as the critical relaxation time. It then follows from the above relations that

$$\tau^+_{cr} = 40. \qquad [1b]$$

The result given by [1b] fairly agrees with the summarized experimental study of McCoy & Hanratty (1977) who suggest a value of $\tau^+_{cr} \simeq 23$, and the deposition measurements of Liu & Agarwal (1974) who observed a value of 30 for τ^+_{cr}. Equation [1b] therefore tends to support the soundness of the present deposition model for $S^+ > 30$.

2.2 *Deposition velocity*

Based on the proposed physical model described above, the analysis for drop deposition is performed for particle $\tau^+ > 40$. The following assumptions are made.

(1) The flow is fully developed.

(2) The concentration profile is developed so that entrance effects are not present.

(3) Drop concentration is small enough to consider that the fluid turbulence characteristics are unaltered.

(4) The size of the particles is small enough that their motion relative to the fluid obeys Stokes law of resistance.

(5) There is no wall rebound or reentrainment of drops, once the drops deposit on the wall. However, when deposition occurs there exists a sticking probability which is related to the adhesion of particles at the immediate vicinity of the wall, even though there is no drifting motion due to field effects (Soo & Tung 1972). The adhesive forces are either electrical or liquid (viscosity and surface tension) in nature. The observed mean deposition velocity therefore includes the probability that some drops do not merge into the wall layer, but rebound and are removed by turbulent diffusion. In the present paper it is assumed that the sticking probability is unity in view of the experimental observations of Cousins & Hewitt (1968).

(6) The local mass flux of droplets in the turbulent core varies linearly from zero at the tube center to a value of N_0 at $y^+ = 30$ and remains constant equal to N_0 in the region $0 < y^+ < 30$. This assumption however requires plug flow idealization. Because the turbulent flow velocity profile is relatively flat, the above approximation introduces little error, as mentioned by Kays (1966).

The rate law for the diffusion of droplets due to concentration gradients is given by

$$N = \epsilon_p \frac{dc}{dy}, \qquad [2]$$

where N is the local rate of mass flux of droplets in the radial direction, c is the concentration of the droplets in the units of mass/volume and ϵ_p is the particle diffusivity equal to $\delta\epsilon_f$, ϵ_f being the fluid eddy diffusivity.

Equation [2] for the particle flux is based on the assumption that the particles in the gas core interact with the turbulent eddies in such a way that particle dispersion is considered essentially diffusive. It has been employed by almost every investigator to date (Friedlander & Johnstone 1957, Namie & Ueda 1973, Forney & Spielman 1974, Rouhiainen & Stachiewicz 1970, Hagiwara *et al.* 1979) in their analyses of deposition process, in view of its adequacy in predicting the trends of the observed deposition data. The underlying basis for such an assumption is that the deposition flux is found to be proportional to the bulk concentration of particles in the gas stream, as analogous to heat and mass transport in single phase flows, suggesting that the mechanism is governed by a gradient-type diffusion. It is in fact pointed out by Lumley (1978, p. 308) that although there is no theoretical support for this consideration, it suggests the use of a diffusion equation.

However, there is an upper limit on the particle size, above which the use of [2] becomes questionable. In the present theory it is inherently assumed that the particle diameter should be smaller than the smallest dynamically significant wavelength present in the turbulence, for example, the Kolmogorov microscale, $\eta = (\nu_G^3/\epsilon)^{1/4}$ where ν_G is the kinematic viscosity of the fluid and ϵ is the dissipation rate per unit mass, given by $\epsilon \sim u^3/l$ where u and l are the velocity and the length scales of the large scale turbulence respectively (Tennekes & Lumley 1972, p. 20).

As is well known the eddy diffusivity of fluid is not constant across the turbulent core (Kays 1966). The eddy viscosity expression proposed by Reichardt (1951) is considered here, and is given by

$$\epsilon_f = \frac{\mathrm{K} r_0 u^*}{6}\left[1-\left(\frac{r}{r_0}\right)^2\right]\left[1+2\left(\frac{r}{r_0}\right)^2\right]. \qquad [3]$$

Here K is the mixing length constant equal to 0.4. Integration of [2] between $y^+ = 30$ and a point y^+ in the core leads to the concentration distribution as

$$c - c_b = \frac{N_0}{u^*\delta K}\left[\ln\left(\frac{1+2x}{1-x}\right)\right]_{x=(1-y^+/r_0^+)^2}^{(1-30/r_0^+)^2}. \qquad [4a]$$

At $y^+ = r_0^+$, we have

$$c_c - c_b = \frac{N_0}{u^*\delta K}\ln\left[\frac{1+2(1-30/r_0^+)^2}{1-(1-30/r_0^+)^2}\right], \qquad [4b]$$

where c_c and c_b are the concentration at the tube center and the edge of the buffer layer respectively. The mass transfer coefficient or the so-called "deposition velocity" is defined as

$$k_d = N_0/\bar{c}, \qquad [5a]$$

with the bulk concentration $\bar{c}$ given by

$$\bar{c} = \int_0^{r_0} cUr\,dr \Big/ \int_0^{r_0} Ur\,dr, \qquad [5b]$$

where $U(r)$ is the radial velocity distribution in the tube.

In evaluating[5b], it is assumed that $c = c_b$ for $o < y^+ < 30$. This has become necessary in view of the uncertainties regarding the phase distribution in the immediate vicinity of the wall. However $\bar{c}$ is seen to be relatively insensitive to the type of concentration profile assumed in the sublayer, as would be expected.

In order to complete the solution of the problem, an additional relation for c_b is necessary. This is obtained here by invoking an auxiliary boundary condition of the form (Beal 1970, Liu & Ilori 1974):

$$N_0 = v_{pb} c_b\,, \qquad [5c]$$

where v_{pb} is taken here as the RMS particle velocity at the edge of the buffer layer, wherefrom free flight is considered to start (Beal 1970). From the relations to be derived later in the text (see [8]), it can be shown that

$$v_{pb} = v_{fb} \surd(\delta)\,, \qquad [5d]$$

where v_{fb} is the RMS radial velocity of the fluid at $y^+ = 30$, which can be taken equal to 0.75 u^*. Therefore

$$v_{pb} = 0.75 u^* \surd(\delta)\,. \qquad [5e]$$

Equating [5a] and [5c], we obtain a relation for c_b as

$$c_b/\bar{c} = k_d/(0.75\,u^* \surd(\delta))\,. \qquad [5f]$$

The value of $\bar{c}/c_c$ is determined from [4b], [5b] and [5f]. In [5b], the following distribution for $U^+(=U/u^*)$ is considered.

$$U^+ = y^+, \qquad 0 < y^+ < 5 \qquad [6a]$$

$$U^+ = 5 \ln y^+ - 3.05, \qquad 5 < y^+ < 30 \qquad [6b]$$

$$U^+ = 5.5 + 2.5 \ln \left[\frac{y^+\{1.5(1 + r/r_0)\}}{1 + 2(r/r_0)^2}\right], \quad 30 < y^+ < r_0^+\,. \qquad [6c]$$

Equation [6c] is due to Reichardt (1951), and is in excellent agreement with the experimental data up to the tube center (Kays 1966).

From [4b], [5a] and [5f], it is shown that the dimensionless deposition velocity is given by

$$\frac{k_d}{u^*} = \frac{(c_c/\bar{c})}{2.5 \ln\left[\dfrac{1 + 2(1 - 30/r_0^+)^2}{1 - (1 - 30/r_0^+)^2}\right] + (\surd(\delta)/0.75)}\,. \qquad [7]$$

2.3 *Particle to fluid diffusivity ratio* δ

The diffusivity of a particle in turbulent flow has been analysed by Tchen (1947). The details and limitations of his theory, and further considerations towards improvement have been given in Soo (1967) and Hinze (1975). For solid particles or water droplets in a gas stream ($\rho_G \ll \rho_d$), in the absence of external forces due to gravity, additive mass and pressure gradient, it can be shown (Hinze 1975, Eq. 5–211; also Mastanaiah 1980) that Tchen's theory for small diffusion time ($t \ll T_{fL}$) leads to

$$\delta = \frac{\epsilon_p}{\epsilon_f} = \frac{\int_0^\infty E_{pL}(n)\,\mathrm{d}n}{\int_0^\infty E_{fL}(n)\,\mathrm{d}n} = \frac{\overline{v_p'^2}}{\overline{v_f'^2}} = \frac{1}{1 + \frac{1}{18}\rho_d \frac{d^2}{\mu_G}\frac{1}{T_{fL}}}, \qquad [8]$$

where v_p' and v_f' denote the fluctuating radial velocity, and $E_{pL}(n)$ and $E_{fL}(n)$ are the Lagrangian energy spectrum functions of particle and fluid respectively, t represents time, and T_{fL} is the Lagrangian integral time scale of the fluid defined as

$$T_{fL} = \int_0^\infty R_{fL}(t)\,\mathrm{d}t. \qquad [9]$$

The Lagrangian correlation coefficient R_{fL} is defined as

$$R_{fL}(t) = \overline{v_f'(t_0)v_f'(t_0+t)} \Big/ v_f^2, \qquad [10]$$

where averaging is done over a number of particles. Equation [8] is similar to that derived by Friedlander (1957). Rouhiainen & Stachiewicz (1970) and Namie & Ueda (1973) have used an expression similar to [8] in their analyses of deposition motion.

The assumption of a small time diffusion leading to [8] for the diffusivity ratio δ can be justified as below. For shear flow through tubes of finite dimensions, the radial diffusion time is of the same scale as the characteristic Lagrangian integral time scale T_{fL} which is of the order of r_0/u^*. In such situations were $t \sim T_{fL}$, the dispersion of the particle can be rather well approximated as a small time diffusion process (Monin & Yaglom 1971, p. 545), due to the fact that the diffusivity is only very weakly dependent on the specific form of the correlation function.

There does not exist sufficient experimental information about the quantity T_{fL} in [8] for turbulent pipe flows because of the difficulty of measurements. It is generally related to the Eulerian integral scale T_E by (see Monin & Yaglom 1971, p. 577):

$$T_{fL} = \beta T_E, \qquad [11]$$

where

$$T_E = l_E/u, \qquad [12]$$

u and l_E being the radial velocity scale and Eulerian length scale of turbulence respectively. For the turbulent core flow in a tube, u can be considered uniform and taken equal to $0.8\,u^*$. Equation [11] is based on the consideration that the Lagrangian and the Eulerian autocorrelations have similar shape but differ only in the time scale.

However, there exist some quantitative discrepancies in the reported measurements of l_E for a pipe flow (Komasawa *et al.* 1974, Martin & Johanson 1965) although the data in general show an increase of l_E with Re. Komasawa *et al.* (1974) have attributed this variation in l_E to the fact that the geometry of the turbulence generating devices has a large effect on the macroscopic turbulence quantities. They further note that no significant difference is seen between the spectra measured at some radial distances from the center in the core region ($0.65 < y/r_0 < 1.0$), thus indicating that the radial variation of l_E may not be appreciable. The measurements of l_E obtained by Martin & Johanson (1965) for water will be adopted here, as they cover a wider range of Re (2×10^4–1.4×10^6), and are represented by

$$l_E/r_0 = 5.028\times 10^{-4}\,\mathrm{Re}^{0.509}\,. \qquad [13]$$

There exists even greater degree of uncertainty in the present state of knowledge concerning the important quantity β which relates the Lagrangian and the Eulerian integral time scales. As pointed out in Monin & Yaglom (1971, p. 577), the reported values of β are extremely scattered about a mean value of $\beta = 4$ (Hay & Pasquill 1957). More recently, Snyder & Lumley (1971) have interpreted a value of $\beta = 3$ based on their measurements in a grid generated turbulence. It should be noted that β is not a universal constant; but in view of the absence of reliable and consistent value of β at the present time, a value of $\beta = 3$ is employed in the present work as a first approximation. Therefore we have

$$T_{fL} = \alpha\beta r_0/u^*\,, \qquad [14a]$$

where

$$\alpha = 6.3\times 10^{-4}\,\mathrm{Re}^{0.509}\,, \qquad [14b]$$

$$\beta = 3\,. \qquad [14c]$$

From [8] and [14], the diffusivity ratio δ is obtained as

$$\delta = \frac{1}{1+\frac{1}{9}\frac{\rho_p}{\rho_G}\left(\frac{d_p}{D}\right)^2 \mathrm{Re}\sqrt{(f/2)}\left(\frac{1}{\alpha\beta}\right)}\,, \qquad [15]$$

where f is the friction factor. Expressed in dimensionless relaxation time, τ^+ defined in [1a], and noting that $r_0^+ = \mathrm{Re}\sqrt{(f/2)}/2$, [15] becomes

$$\delta = \frac{1}{1+\frac{2}{\mathrm{Re}\sqrt{(f/2)}}\tau^+/(\alpha\beta)} = \frac{1}{1+(\tau^+/r_0^+)/(\alpha\beta)}\,. \qquad [16]$$

The friction factor is calculated from the well-known correlations for smooth tubes as

$$\begin{aligned} f &= 0.0791\,\mathrm{Re}^{-0.25}, &\quad 3\times 10^3 < \mathrm{Re} < 10^5 \text{ (Blasius)}\\ &= 0.046\,\mathrm{Re}^{-0.2}, &\quad 10^5 < \mathrm{Re} < 10^6\,. \end{aligned} \qquad [17]$$

Thus [16] shows that δ depends on τ^+ and Re only. The trend of δ given by [16] is similar to that obtained recently by Soo (1978) who has deduced three different expressions for the diffusivity ratio depending on the particle size. However, the present result [16] offers the advantage of being continuous for all τ^+ within the Stokes regime.

It may be remarked that δ as given by [16] is spatially uniform across the core. This appears to be reasonable since the radial intensity of fluid turbulence, v_f, which is responsible for the particle radial migration, is nearly constant in the core region (Hinze 1975, pp. 725 & 726), and since the Eulerian length scale as mentioned earlier is also nearly independent of radial position. In fact, [8] suggests that the ratio δ can be computed as a function of y^+ with a knowledge of the radial distribution of the energy spectrum functions. Detailed calculations by Namie & Ueda (1973) incorporating the limited Eulerian data of Comte-Bellot (1965) as an approximation to the Lagrangian energy spectrum shows that the variation of the diffusivity ratio is not appreciable in the turbulent core.

It is seen from [16] that δ becomes 1 at $\tau^+ = 0$, and tends to zero as $\tau^+ \to \infty$. The result is in agreement with the observation that very large particles are not influenced by the eddy motion and remain entrained near the tube center (see Cousins & Hewitt 1968). However, it is to be noted that for particles with $\mathrm{Re}_d = d|v_p - v_f|/\nu_G > 1.0$, the linear resistance law due to Stokes becomes less accurate. The limitation of the present model due to the assumption of Stokes law (see [20] will be discussed later in the text. From [7] and [16] the final expression for the deposition velocity is obtained in dimensionless form as

$$\frac{k_d}{u^*} = \frac{c_c/\bar{c}}{\left\{2.5 \ln\left[\frac{1+2(1-30/r_0^+)^2}{1-(1-30/r_0^+)^2}\right] + \frac{1}{0.75}\sqrt{\left(\frac{1}{1+(\tau^+/r_0^+)/(\alpha\beta)}\right)}\right\}} \cdot \frac{1}{[1+(\tau^+/r_0^+)/(\alpha\beta)]}, \qquad [18]$$

where $\alpha = 6.3 \times 10^{-4}\,\mathrm{Re}^{0.509}$, and $\beta = 3$, as given by [14b] & [14c] respectively. The sensitivity of k_d/u^* to β in [18] will be considered in section 5, in view of the uncertainty in the value of β.

Equation [18] above suggests that the dimensionless deposition velocity k_d/u^* is a function of Re and τ^+ only.

On the other hand, according to Friedlander & Johnstone (1957), for $S^+ > 30$ the deposition velocity is given by

$$k_d/\bar{U} = f/2, \qquad [19a]$$

which can be written as

$$k_d/u^* = \sqrt{(f/2)} = 2r_0^+/\mathrm{Re}. \qquad [19b]$$

Equation [19b] implies that k_d/u^* is independent of particle size but depends on Re only. The improvement of the present result [18] over [19b] will be illustrated later in the text.

In many practical situations, the suspended droplets may not be of uniform size but have a size distribution. In such cases an appropriate mean diameter has to be specified for evaluating τ^+. The arithmetic mean drop size is taken as the effective droplet diameter in the present calculations. Although most of the mass is carried by larger drops, an arithmetic mean diameter is chosen here for the following reason: [18] suggests that the deposition velocity varies as $1/d^2$ for larger drops, and since the mass of the drops varies as d^3, the length mean diameter appears to be a reasonable characteristic drop size in the calculation of deposition flux $N_0 = k_d\bar{c}$. In other words, the probability of larger drops being deposited is relatively small compared to that of the small drops, because larger drops are less influenced by the eddy motion. This is consistent with the fact that the radial transport of small particles is associated with the momentum exchange with the turbulent eddies, in which case, as mentioned by Soo (1967), a length mean diameter d_{10} would be appropriate.

It should be recognized that the present results cannot be applied to arbitrarily large τ^+ in view of the restriction on the Stokes resistance law, which is valid for $\mathrm{Re}_d < 1.0$. In fact there exists a value of $\tau^+ = \tau_m^+$ above which the condition that $\mathrm{Re}_d < 1.0$ will not be satisfied. By definition,

$$\mathrm{Re}_d = d|v_p - v_f|/\nu_G. \qquad [20a]$$

In [20a], v_f is taken as 0.8 u^* and v_p is given by [8] as

$$v_p = v_f \sqrt{(\delta)}\,, \tag{20b}$$

where δ is given by [16]. After some algebraic manipulation, it can easily be shown that the maximum value of τ^+ is given by the relation

$$\mathrm{Re}_d^* = 0.75\sqrt{(18\tau_m^+(\rho_G/\rho_d))}(1-\sqrt{(\delta)}). \tag{20c}$$

In [20c], Re_d^* is the value of Re_d above which Stokes law will be inaccurate, and taken equal to 1.0. Thus it is seen that

$$\tau_m^+ = f(\mathrm{Re}, \rho_d/\rho_G)\,. \tag{20d}$$

It is seen from [20c] that τ_m^+ increases with Re and ρ_d/ρ_G. For instance, for water dropletes in air at atmospheric pressure (ρ_d/ρ_G is about 750), [20c] suggests that τ_m^+ increases from 249 to 1201 as Re is increased from 10^4 to 10^5. Unfortunately, a decrease in ρ_d/ρ_G results in a decrease in τ_m^+. However, it is important to recall at this point that the present theory can not be applied to relatively small ρ_d/ρ_G, since the derivation of [8] assumes that $\rho_d \gg \rho_G$. This limit on usefulness of τ^+ is to be noted in interpreting the subsequent results of this paper.

3. PREDICTIONS OF DROPLET CONCENTRATION

With the use of [4a] and [5f], the dimensionless radial concentration distribution c/c_c is depicted in figure 2. For purposes of clarity, the profile in the region $0 < y^+ < 30$ is not shown. The results show that the droplet concentration varies considerably across the turbulent core, as opposed to the turbulent velocity profile which is relatively flat in the core region. In general, at any radial location the particle concentration decreases with increasing τ^+. It is also evident from figure 2 that with an increase of Re, the concentration profile becomes less dependent on τ^+. At $\mathrm{Re} = 10^5$, the difference in concentration profile for $\tau^+ = 40$–10^4 is unobservably small. On the other hand, at $\mathrm{Re} = 5\times10^3$, the value of c_b for $\tau^+ = 40$ is more than four times that

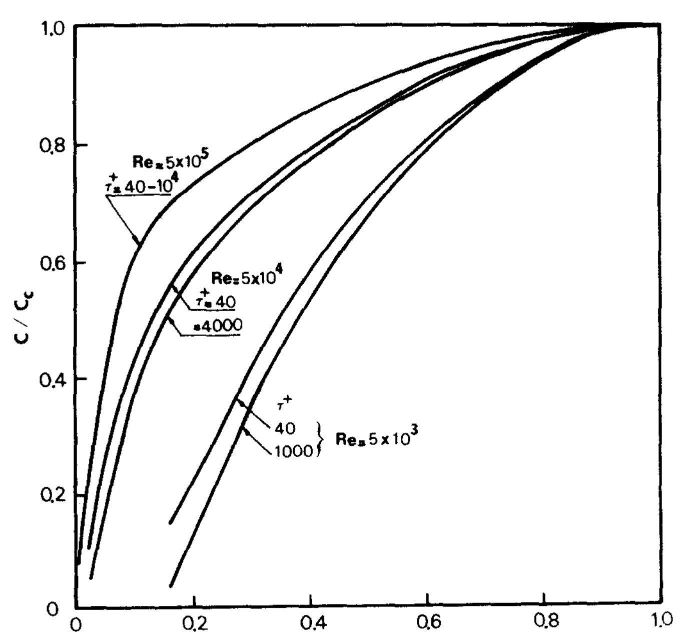

Figure 2. Concentration distributions.

corresponding to $\tau^+ = 1000$. The relatively low sensitivity of the concentration distribution to τ^+ is attributed to the present consideration that turbulent diffusion is important down to $y^+ = 30$ regardless of S^+. This is also evident by an examination of [4a] and [4b].

Equation [5f] shows that the dimensionless concentration at the edge of the buffer layer, c_b/c_c, is of the order of k_d/u^*. Calculations have shown that the value of c_b is almost always less than about 10 per cent of the center line concentration, as also evident from figure 2. The present information regarding c_b is believed to be useful in the understanding of deposition motion, since there appears to be no reported measurements of c_b due to the difficulty of measurements.

The most important conclusion that can be drawn from the results shown in figure 2 is that for large particles ($S^+ > 30$) the major resistance to particle transport resides in the turbulent core. In other words, the dispersion in the bulk is playing a more important role than is generally thought, in governing the resistance characterizing the rate of deposition from turbulent streams. This is in contrast with the trend for the small particles with $S^+ < 30$, since the resistance to transport of small particles characterized by $0.15 < \tau^+ < 40$ (see figure 1) is known to be controlled by the viscous boundary layer, where the stopping distance concept of Friedlander & Johnstone (1975) is appropriate.

4. COMPARISON WITH EXPERIMENTAL DATA

The accuracy of the proposed theory is now tested by comparing the calculated results with the existing experimental data as well as the present measurements.

4.1 *Data for concentration distributions (Hagiwara et al. 1979)*

To the authors' knowledge, the data available for the distribution of droplet concentration in a vertical tube are those recently reported by Hagiwara *et al.* (1979). The data of Hagiwara *et al.* (1979) for concentration distribution at Re = 40,000 is presented in dimensionless form in figure 3. The data shown pertain to two streamwise locations in the test section, i.e. at locations $x = 1000$ and 1200 mm downstream of the nozzle inlet, where entrance effects may not be present. The data are taken down to $y/r_0 = 0.11$ from the tube center. The measured arithmetic mean

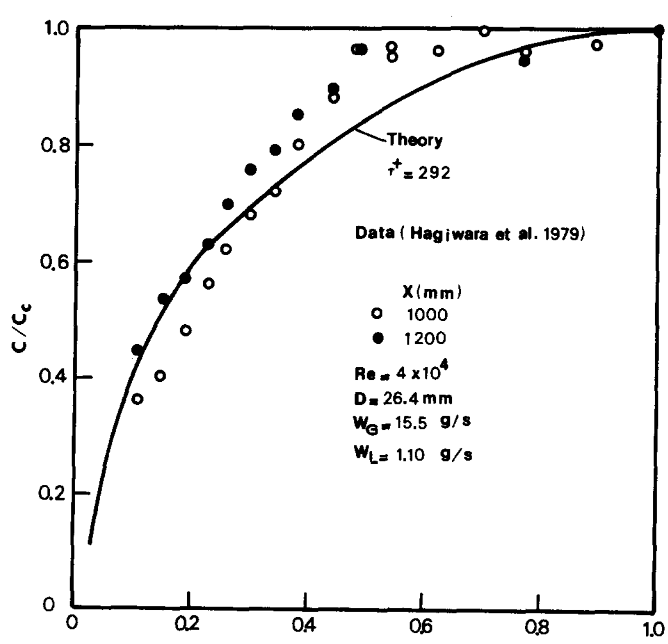

Figure 3. Comparison of predicted concentration profile with the data of Hagiwara *et al.* (1979). Data: Re = 10^4; ○, $X = 1000$ mm; ●, $X = 1200$ mm.

diameter d_{10} is about 32 μm. The corresponding value of τ^+ based on d_{10} is 292. The predicted values of c/c_c from the present analysis is shown in figure 3 for $\tau^+ = 292$.

It is seen from figure 3 that the predicted concentration distribution is in good agreement with the data, except for some region near the tube center where the measured concentration is relatively flat. It is unfortunate that no data appear to exist for concentrations near the periphery of the turbulent core, with which to compare the present predictions. The buffer layer for the conditions of the data presented in figure 3 corresponds to $y/r_0 = 0.0284$. The data however clearly suggest that the primary resistance to particle transport exists in the turbulent core region.

4.2 *Deposition data for monodisperse particles*

Much of the deposition data in the literature are obtained using monodisperse particles. Such data are not associated with uncertainties in particle size and therefore form an excellent choice for comparison with theory. As can be seen from figure 1, bulk of this data for uniform-size particles is limited to particle τ^+ less than about 40, except those of Forney & Spielman (1974), Liu & Agarwal (1974) and Agarwal (1975), while the present analysis is applicable for $\tau^+ > 40$. The data of Forney & Spielman (1974) is however quite scattered (see figure 1) with poor reproducibility, and do not seem to be a good choice for comparison with theory. For monodisperse particles, the present theory could then be compared with the reliable data of Agarwal (1975) over wider range of Re and τ^+, and with a few data points of Sehmel (1968) and Liu (1971) within a narrow range of τ^+ between 40 and 60.

4.2.1 *Deposition data of Agarwal* (1975). Agarwal (1975) has obtained deposition data for uniform-size uranine-tagged olive aerosol in vertical down-flow of air in 3.27-mm i.d. glass tube with $L/D = 91.7$ at Re = 6000, and in 13.8-mm i.d. copper tube with $L/D = 73.9$ at Re = 50,000. The test section tubes are smooth. The drops are generated by means of a vibrating orifice monodisperse aerosol generator. The maximum size of the droplet used is 21 μm, and the droplet to fluid density ratio was about 713. The deposition velocity is determined from the amount of aerosol deposited on the deposition section only, and hence no entrance effects are present.

The calculated deposition velocity k_d^+ vs τ^+ using the present analysis is compared in figure 4 with the above data. The data for Re = 6000 covers a maximum τ^+ of 291 corresponding to $d = 21$ μm, while a maximum of $\tau^+ = 449$ corresponding to $d = 17.9$ μm is obtained for the conditions at Re = 50,000. The data for $\tau^+ > 40$ only are presented in the figure, since that represents the scope of the present work. The experimental curve of Liu & Agarwal (1974) at Re = 50,000 for 12.7-mm i.d. glass tube passes through the data of Agarwal (1975) for the same Re and range of τ^+, and are therefore not separately shown in figure 4. The calculated results from the theory of Friedlander & Johnstone (1975) are also displayed.

It is seen that the theory of Friedlander & Johnstone (1957) is unable to predict the deposition velocity, while the present theory is able to represent the trends of the data, although the theory slightly under-predicts the measurements. The minor deviation between the theory and the data can be partly attributed to the uncertainties associated with the values of l_E and β in [13] and [14c] respectively. Nevertheless the agreement should be considered satisfactory at the present time in view of the apparent complexity of the phenomena involved.

The calculated value of particle Reynolds number Re_d at $\tau^+ = 291$ for Re = 6000 is 1.8, while that at $\tau^+ = 449$ for Re = 50,000 is 1.1. This implies that the present theory appears to apply for Re_d as large as about 2.0. That is, the theory is seen to be satisfactory for particle size having τ^+ somewhat greater than τ_m^+ given by [20c]. This is to be expected since Stokes drag coefficient is not very much in error for Re_d even up to about 2.0, while a value $\mathrm{Re}_d^* = 1.0$ is used in [20c].

4.2.2 *Deposition data of Sehmel* (1968) *and Ilori* (1971). Sehmel (1968) has obtained deposition data in upward vertical flow of air and methylene blue aerosol in 71- and 29-mm i.d. tubes. The droplet to fluid density ratio is about 1163. The maximum particle diameters used are 28

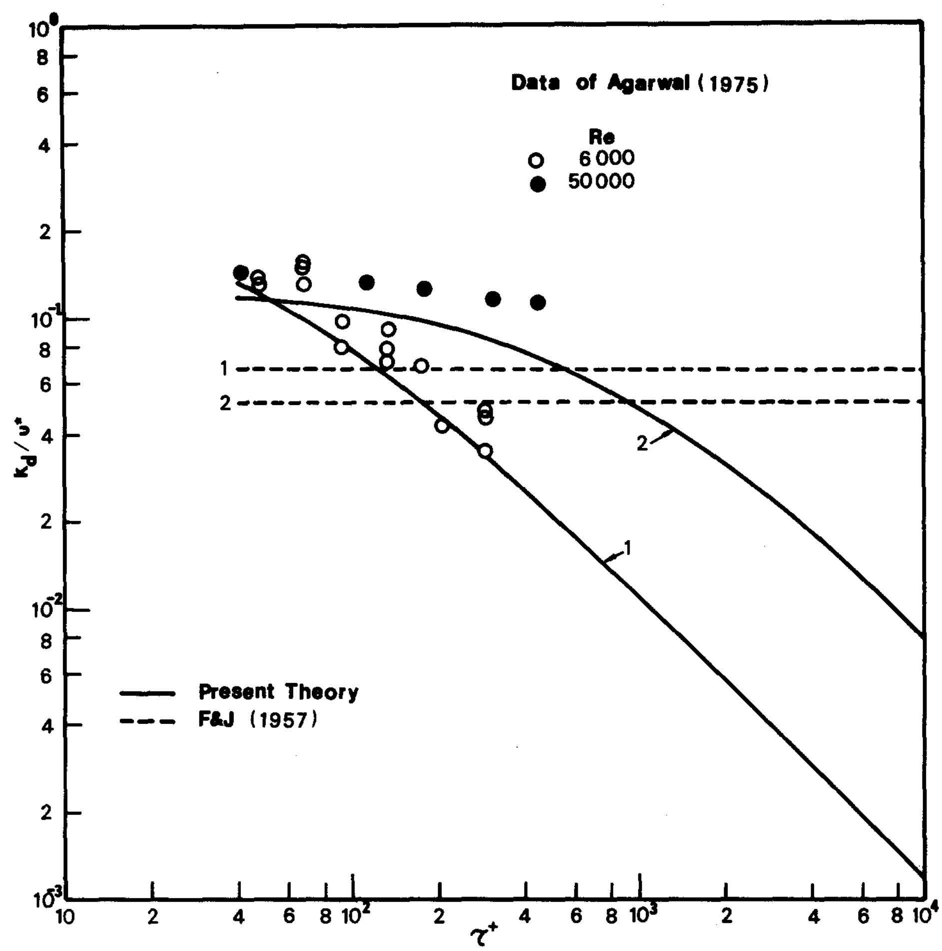

Figure 4. Comparison of predicted deposition velocity with the experimental data of Agarwal (1975). Data: ○ 3.27 mm i.d. glass tube, Re = 6000; ●, 13.8 mm i.d. copper tube, Re = 50,000. Prediction: 1, Re = 6000; 2, Re = 50,000; —, present; ---, Friedlander & Johnstone (1957).

and 8.5 μm for the larger and the smaller diameter tubes respectively. The data of Ilori (1971) are obtained in upward vertical flow of air and 80 per cent methylene–20 per cent uranine in a 29.8 mm vertical tube at Re = 50,000. The droplet to fluid density ratio is about 1317, and the size of the droplets is in the range of 6–9 μm.

The present theory and the theory of Friedlander & Johnstone (1957) are compared in Table 1 with the data of Sehmel (1968) and Ilori (1971). It is evident that the present theory is in satisfactory agreement with the data, while the theory of Friedlander & Johnstone (1957) considerably underpredicts the data. The particle Reynolds number Re_d is about 0.05 or less in all the cases considered in table 1, and is within the Stokes regime.

4.3 *Deposition data of Cousins & Hewitt* (1968)

Cousins & Hewitt (1968) have measured deposition velocities in vertical upflow of air and water in 9.525- and 31.8-mm i.d. tubes. Water is introduced as an annulus through a porous sinter, and entrainment of water droplets is generated due to vapor shear. The measured mean Sauter diameter, d_{32}, ranges from 40 to 70 μm in the smaller tube, and 70 to 110 μm in the larger diameter tube. The d_{10} values used in the present predictions are computed from the average of measured d_{32} and using the relationship $d_{32}/d_{10} = 4.667$, as developed by Tatterson *et al.* (1977). This relationship is based on drop size distributions measured by many investigators.

Table 2 depicts the comparison of calculated deposition velocities with the measured values for the two tube diameters. The data shown in Table 2 for the 9.525-mm dia. tube are averaged for the different test section lengths excluding the shortest section for which entrance effects

Table 1. Comparison of predicted deposition velocities with the experimental data of Sehmel (1968) and Ilori (1971)

Author	D, mm	d_p, μm	Re	τ^+	k_d/u^*			% Error		Re_d
					Expt.	Present Theory	F&J*	Present Theory	F&J	
Sehmel (1968).	71	28.0	35,000	40	·16	·123	·054	23.1	66.2	.05
	29	8.5	60,000	55	·12	·110	·050	8.3	58.8	.04
Ilori (1971)	29.8	9.0	50,000	48	·12	·114	·051	5.	57.5	.05

*F&J – Theory of Friedlander & Johnstone (1957).

Table 2. Comparison of predicted deposition velocities with the experimental data of Cousins & Hewitt (1968)

D, mm	Re	τ^+	k_d, m/s			% Error		Re_d
			Expt.	Present Theory	F&J*	Present Theory	F&J	
9.525	37,380	164	.165	.178	.098	7.9	40.6	.54
	56,051	334	.186	.218	.133	17.0	28.6	.85
31.8	1.4×10^5	354	.137	.129	.070	5.8	46.8	.36
	1.96×10^5	642	.179	.159	.091	10.9	42.6	.54

*F&J – Theory of Friedlander & Johnstone (1957) are observed, and also for the different liquid loadings. The measured deposition velocity for the 31.8-mm pipe, however, corresponds to a deposition length of 1.94 m. No reentrainment of droplets is observed for the data selected here for comparison. It is seen from table 1 that the present theory is in good agreement with the data. The maximum error for the lower Re is about 17 per cent. At higher Re, the error is about 11 per cent. On the other hand, the theory of Friedlander & Johnstone (1957) is in error by about 40 per cent compared to the data. It is interesting to note from table 2 that the particle Re_d has not exceeded about 0.85, thus indicating the validity of the present theory.

The fractional deposition curve for 9.525-mm dia. tube at Re = 37,400 are also compared as shown in figure 5. Data for different liquid flow rates are presented. As is to be expected, good agreement is noted between the predicted curve and the observed data for fractional deposition F given by

$$F = 1 - \exp[-4(k_d/\bar{U})(L/D)]. \quad [21]$$

This relation for F can be obtained by a mass balance. The underprediction of the data at $L/D = 16$ is perhaps due to the entrance effect because of the short length to diameter ratio, while the theory considers only fully developed conditions which occur at relatively large L/D. Figure 6 depicts the fractional deposition curve for 31.8-mm dia. tube at Re = 196,000. Again the agreement is good as in the case of Re = 37,400. It is however noted from figures 5 and 6 that the theory of Friedlander & Johnstone (1957) considerably deviates from the fractional deposition data, as is to be expected because of large error in the predicted value of k_d.

Although the experiments of Cousins & Hewitt (1968) and of Agarwal (1975) appear to differ in the method of droplet generation and dispersion into the flow, the measured deposition velocities could not be affected because the measurements correspond to large distances from the inlet where entrance effects are not present, as mentioned earlier.

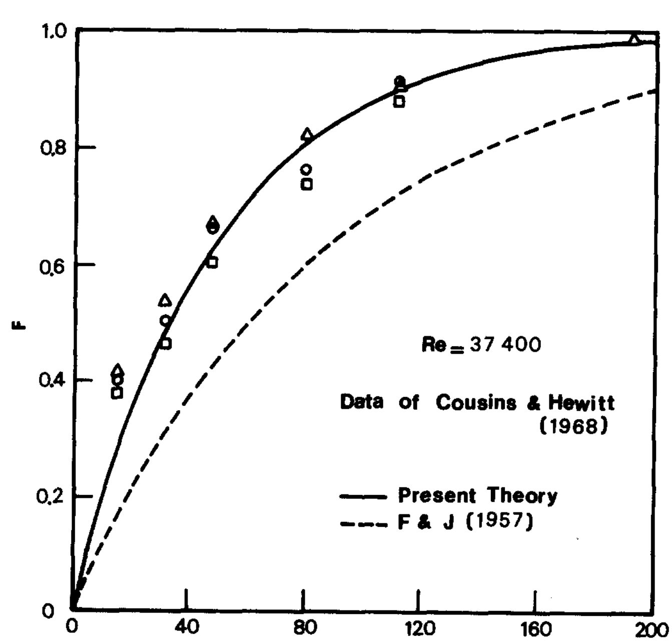

Figure 5. Comparison of theory with data for fractional deposition in 9.525 mm i.d. tube at Re = 37,400. Data of Cousins & Hewitt (1968): gas flow rate $W_G = 18.2$ kg/h; liquid loading W_L; □, 22.7 kg/h; ○, 34 kg/h; △, 45.3 kg/h. Prediction: ——, present, $d = 12$ μm; ---, Friedlander & Johnstone (1957).

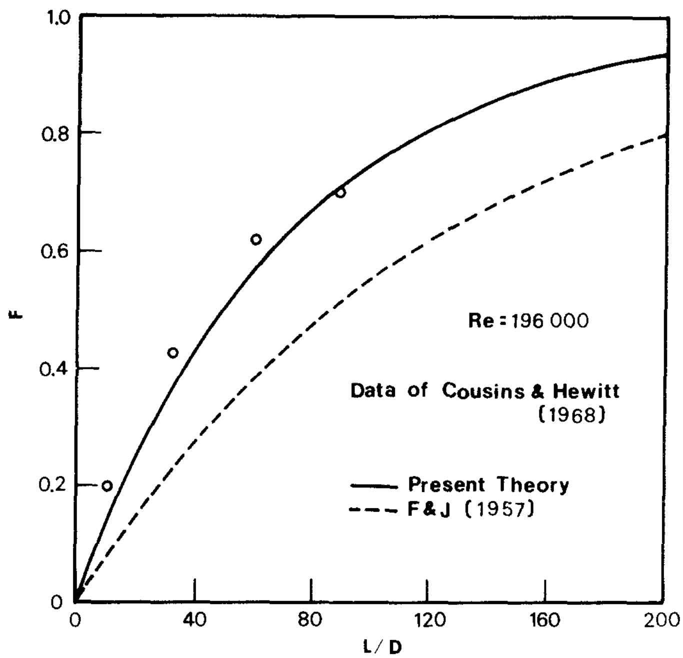

Figure 6. Comparison of theory with data for fractional deposition in 31.8 mm i.d. tube at Re = 196,000. Data of Cousins & Hewitt (1968): gas flow rate $W_G = 318.0$ kg/h; liquid loading $W_L = 72.7$ kg/h. Prediction: ——, present, $d = 19\ \mu$m; ---, Friedlander & Johnstone (1957).

4.4 *Present deposition measurements*

4.4.1 *Experimental apparatus and procedure.* In order to provide a further check on the proposed deposition model, the theory is also compared with the present deposition data. The authors have recently obtained deposition rate measurements for air–water system in a vertical tube at near atmospheric pressure. Details of the experiment are given in Mastanaiah (1980). Air from a compressor passes first through a refrigerated air dryer to remove moisture, and later through an oil filter to eliminate oil content. The liquid droplets are generated by an atomizer with a sharp-edged air orifice and a cylindrical liquid nozzle, the dimensions of which are similar to those employed by Nukiyama & Tanasawa (1938, 1939). Secondary air is used for the atomizer, while the primary air stream is used to vary the bulk flow rate in the test section. The Sauter mean diameter of the droplet size distribution d_{32} is calculated from the Nukiyama & Tanasawa correlation (see Nukiyama & Tanasawa 1938, 1939). Preliminary measurements of drop size using a glass plate coated with silicone oil for droplet catching and placed at the exit of the acrylic tube have shown that the measured d_{32} is in good agreement with that given by the above mentioned correlation. The values of d_{10} are obtained from d_{32}, using the relations for the drop diameter ratios developed by Tatterson *et al.* (1977). It is possible to vary drop size and droplet concentration by adjusting the secondary and the primary air flow rates.

A schematic of the experimental system for the deposition study is shown in figure 7. An entrance section having 12.7 mm i.d. and L/D of 60 is employed to insure that the flow is fully developed at the inlet of the test section. The test section is a smooth acrylic tube 12.7 mm i.d. and 889 mm long. Extraction sections are used to remove the liquid film formed on the wall at the two ends of the test section.

The technique of measuring the deposition rate is similar to that applied by Cousins & Hewitt (1968). During the experiment, the liquid layer formed at the inlet of the test section is completely removed. The drops then migrate towards the wall and form a thin liquid film that grows continuously along the test section. The liquid film formed at the test section exit is also removed completely. The flow rate of liquid collected at the test section exit gives the amount of liquid deposited over the entire test section length, and hence the fractional deposition F. With a knowledge of F, the deposition velocity k_d is determined from [21].

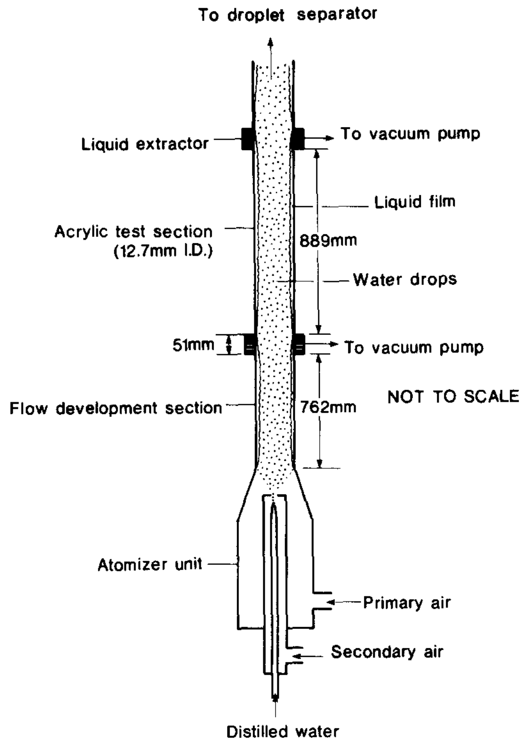

Figure 7. Experimental system for deposition studies.

During the experiments, visual observation has indicated that the test section wall is covered with a thin liquid film usually smooth but sometimes with small ripples at the larger water flow rates. However no significant wave on the water film surface has been noted so that the droplet entrainment (generation of droplets and their dispersion into the core from the liquid layer on the wall) is believed not to have occured in the experiments. The entrainment E is defined as the amount of liquid entrained expressed as a fraction of the liquid film flow rate. The absence of entrainment for the present experimental conditions is confirmed using the correlation of Truong Quang Minh & Huyghe (1965), which suggests entrainment E of less than 2 per cent for Re = 94,600, and less than 0.5 per cent for Re = 52,500. In view of these factors the entrainment effect is not taken into account in deducing the deposition velocities. The flow rate of air removed through the extractor is however measured to be less than about 1 per cent of the total air flow rate through the test section, and is therefore considered to have no significant effect on the measured deposition data. An error analysis has indicated that the measured deposition velocities are within about 12 per cent accuracy. The reproducibility of the data has been assured by repeating some of the runs. The data in general are consistent without significant scatter and are therefore considered reliable.

4.4.2 *Comparison of theory with present data.* The measured dimensionless deposition velocities for Re = 52,500 and 94,600 are depicted in figures 8 and 9 respectively. For the data at Re = 52,500, the test section inlet pressure is 1.15 bar, the drop diameters d_{10} are varied from 12 to 46 μm and the droplet concentration is in the range of $(8.5\text{–}62)\times 10^{-3}$ kg/m^3. The corresponding dimensionless drop relaxation time τ^+ ranges from 264 to 3520. The conditions for Re = 94,600 are as follows: the test section inlet pressure is 1.37 bar, the d_{10} values range from 8 to 45 μm, and the droplet concentration is varied from $(22 \text{ to } 75)\times 10^{-3}$kg/m^3, while τ^+ has a range of 295–8670. The room temperature is about 295 K. It is interesting to note that the present data pertain to relatively large τ^+ compared to those of Agarwal (1975) and of Cousins & Hewitt (1968) which are limited to τ^+ of about 40–640, while the present measurements yield

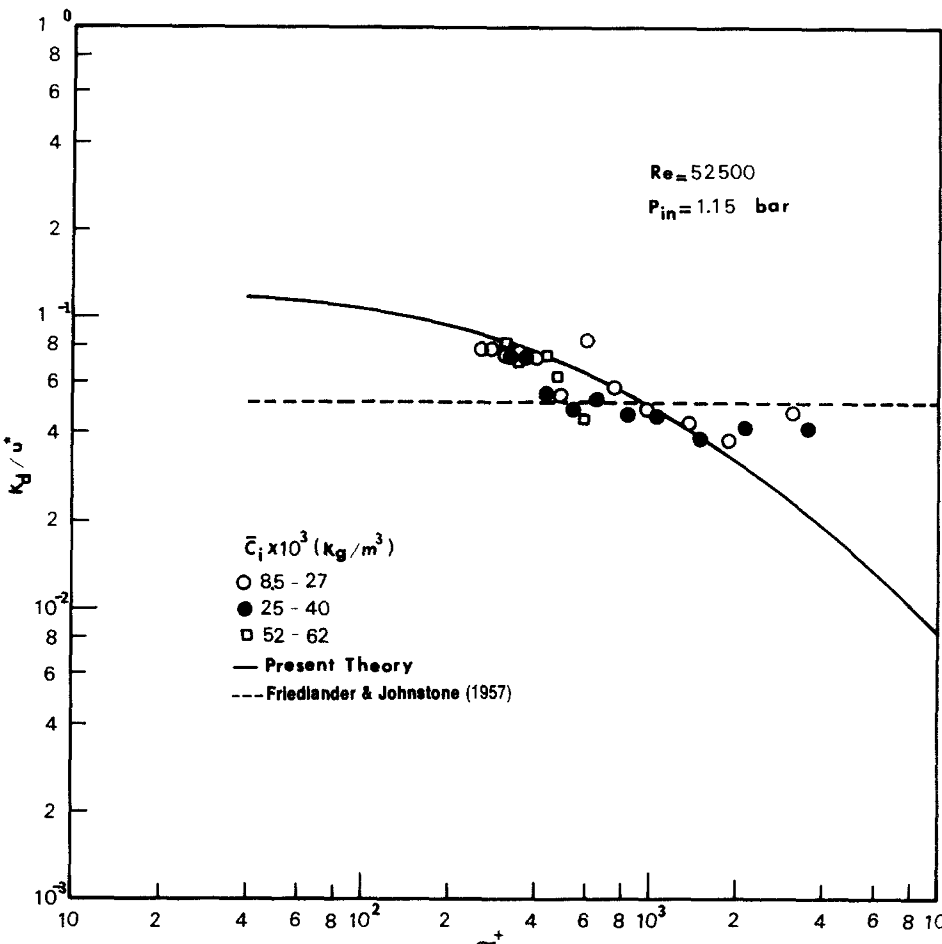

Figure 8. Comparison of predicted deposition velocity with the present experimental data for 12.7 mm i.d. acrylic tube at Re = 52,500: gas flow rate $W_G = 9.525 \times 10^{-3}$ kg/s; liquid loading W_L: O, 1.034×10^{-3} kg/s; ●, 1.751×10^{-3} kg/s; □, 2.501×10^{-3} kg/s.

data for τ^+ in the range of 260–8700. Therefore the present data are believed to be a valuable supplement to the existing body of deposition data.

It is seen from figures 8 and 9 that k_d/u^* decreases with τ^+ up to some value of τ^+ beyond which it remains nearly independent of τ^+. This appears to be an important new information since this trend has not yet been reported earlier. The results also reveal that k_d/u^* is not very sensitive to Re. Within the range of droplet concentration employed in the present experiments, the dependence of k_d/u^* on the droplet concentration is not found to be significant. This is a characteristic trend at low droplet concentrations ($\bar{c}$ typically less than about 1.0 kg/m^3) as observed earlier by Cousins & Hewitt (1968) for a vertical system. At higher concentration, k_d/u^* is generally found to decrease (Namie & Ueda 1972) due to changes in the turbulence characteristics of the gas phase. Particle interactions also become important above certain concentrations (see Batchelor & Green 1972). However, the consideration of higher concentrations is outside the scope of our present work which is concerned only with sufficiently dilute suspensions.

The predictions of k_d/u^* from the present theory and the theory of Friedlander & Johnstone (1957) and also compared with the data in figures 8 and 9. It is observed from figure 8 that the present theory is in good agreement with the data for Re = 52,500 up to τ^+ of about 1500 above which the theory underpredicts the data, the deviation increasing with increasing τ^+. Near the point of deviation, the value of Re_d is about 3.1. Similar trend is noticed for Re = 94,600 also. Figure 9 suggests that the present analysis agrees with the data for Re = 94,600 up to τ^+ of 2000

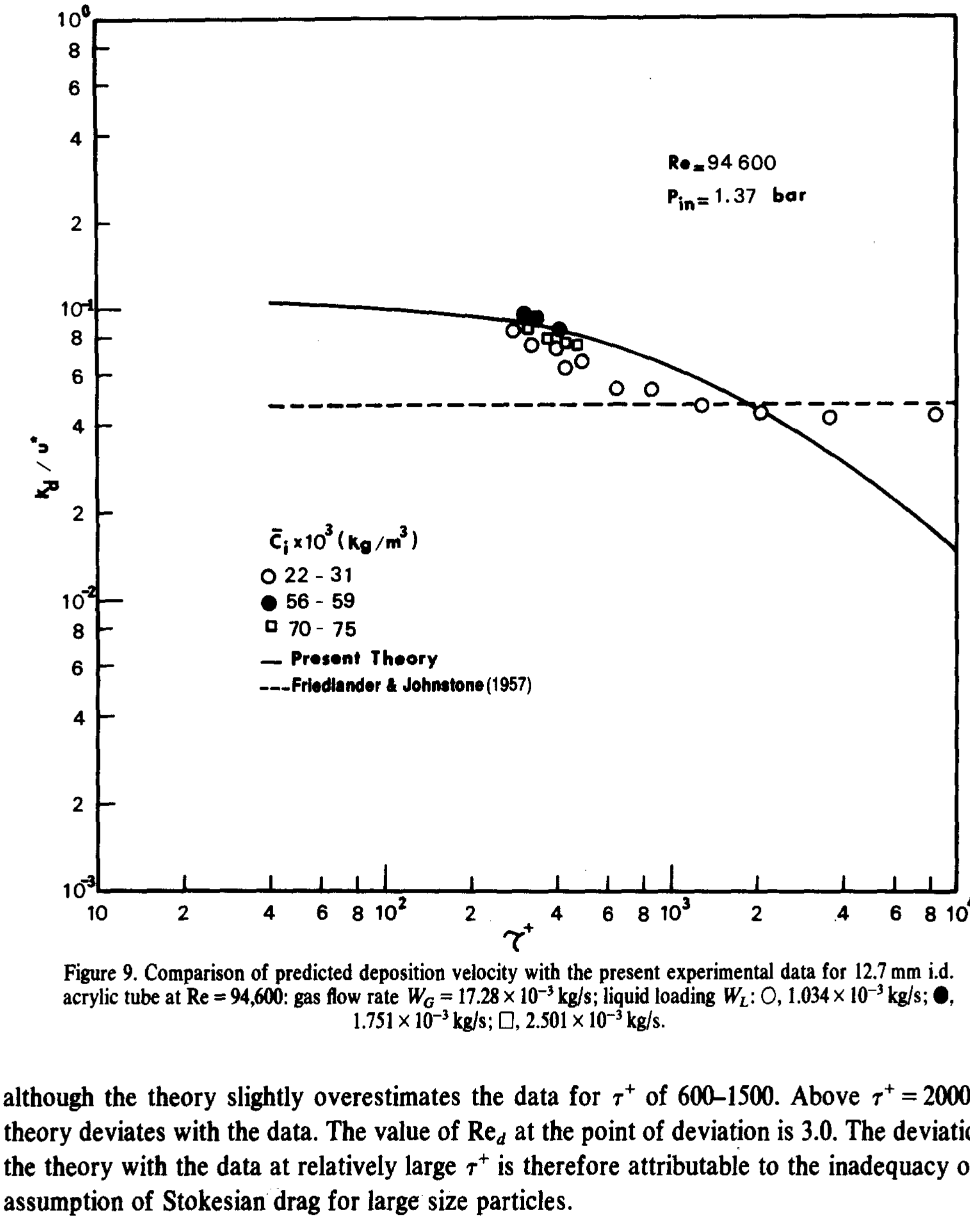

Figure 9. Comparison of predicted deposition velocity with the present experimental data for 12.7 mm i.d. acrylic tube at Re = 94,600: gas flow rate $W_G = 17.28 \times 10^{-3}$ kg/s; liquid loading W_L: O, 1.034×10^{-3} kg/s; ●, 1.751×10^{-3} kg/s; □, 2.501×10^{-3} kg/s.

although the theory slightly overestimates the data for τ^+ of 600–1500. Above $\tau^+ = 2000$, the theory deviates with the data. The value of Re_d at the point of deviation is 3.0. The deviation of the theory with the data at relatively large τ^+ is therefore attributable to the inadequacy of the assumption of Stokesian drag for large size particles.

On the other hand figure 8 shows that for Re = 52,500, the theory of Friedlander & Johnstone (1957) is not able to represent the trend of the data up to τ^+ of about 400, but there is seen to be a surprisingly good agreement for τ^+ of 400–3000. Similar trend is observed for Re = 94,600 also. However, since their theory is not able to describe the deposition velocity for $\tau^+ < 400$, the seemingly good comparison obtained for $\tau^+ > 400$, as in the case of Re = 52,500, does not seem to have significance.

In view of all the above comparisons, it is evident that the present theory offers a considerable improvement over the theory of Friedlander & Johnstone (1957).

5. DISCUSSION

In the present theory, the relaxation time is based on Stokesian drage only. It is true that for large sized particles Stokes law is probably not valid. However, in all the experimental data of other investigators, with which the present theory is compared, the particle Reynolds number Re_d based on the relative velocity has not exceeded 1.8. Hence it is evident that the application of Stokes drag law has not diminished the reliability of the present analysis insofar as the range of variables of the aforementioned experimental data are concerned. For larger particles, the application of the present result should be viewed with caution as demonstrated by the present data. Examination of [20c] further suggests that outside the Stokes regime, the ratio ρ_d/ρ_G may appear as an additional parameter.

A more stringent limitation on particle relative velocity is due to the assumption in Tchen's theory that during the motion of the particle the neighborhood will be formed by the same fluid, i.e. no overshooting. The condition is that the relative velocity must be smaller than the characteristic velocity $(\epsilon \nu_G)^{1/4}$ (see Hinze 1975, pp. 462–463); i.e.

$$|v_p - v_f| \lesssim (\epsilon \nu_G)^{1/4}. \qquad [22a]$$

By definition of Re_d and η, [22a] leads to

$$\mathrm{Re}_d \lesssim \frac{d}{\eta}. \qquad [22b]$$

Calculations by the authors have shown that the particle diameters employed in the data of others considered for comparison in the present paper satisfy the requirement that the particle size is smaller than the Kolmogorov microscale, thereby justifying the usefulness of the present results. In the calculation of η, u is taken as 0.8 u^* and l is given by [13]. For instance, for the data of Agarwal (1975) at Re = 50,000, the maximum particle diameter used is 18 μm, while the microscale is 22.5 μm. For the experiment of Cousins & Hewitt (1968) at Re = 196,000, the characteristic d_{10} is 19 μm and the corresponding microscale is 25 μm, for which case, $d_{10}/\eta = 0.76$ and $\mathrm{Re}_d = 0.54$ (see table 2), thus justifying the condition [22b]. It appears however that the length scale restriction justifying a continuum approximation is somewhat more stringent than the Stokes law requirement in the kind of flows of interest in the present study.

Regarding the sensitivity of the final result to the value of β in [18], it can be shown to some approximation that

$$k_d \sim \frac{\beta}{\beta + \tau^+/(r_0^+\alpha)}. \qquad [23]$$

It is seen from the above equation that for relatively small values of τ^+/r_0^+, the deposition velocity is weakly dependent on β. At large value of τ^+/r_0^+, $k_d \sim \beta$. At relatively large τ^+, the present theory has a limitation due to the assumption of Stokesian drag law. Therefore, within the range of τ^+ for which the present theory can be applied, the deposition velocity is not very sensitive to β. For example, using the data from table 2, it is found that when β is decreased tenfold from 3.0 to 0.3, the deposition velocity decreases by a factor of 3.4 for Re = 37,400 and 2.1 for Re = 1.96×10^5 at the largest τ^+ considered. As more consistent and accurate measurements of β become available, the improved value of β can be accomodated in the solution.

6. CONCLUSION

It has been shown that the dispersion in the core plays a principal role in governing the resistance characterizing the rate of deposition of large sized particles from a turbulent gas stream. The proposed expression for the deposition velocity, given by [18], is believed to be of considerable practical application in predicting droplet deposition rates in two-phase flows.

Acknowledgement—This work was supported by the National Science Foundation under Grant ENG 78-06211.

REFERENCES

Agarwal, J. K. 1975 Aerosol sampling and transport. Ph.D. Thesis, Univ. of Minnesota, Particle Tech. Lab. Publ. 265.

ALEXANDER, L. G. & COLDREN, C. L. 1951 Droplet transfer from suspending air to duct walls. *Indust. Engng Chem.* **43**, 1325–1331.

BATCHELOR, G. K. & GREEN, J. J. 1972 The hydrodynamic interaction of two small freely-moving spheres in a linear flow field. *J. Fluid Mech.* **56**, 375–400.

BEAL, S. K. 1970 Deposition of particles in turbulent flow on channel or pipe walls. *Nucl. Sci. Engng* **40**, 1–11.

CLEAVER, J. W. & YATES, B. 1975 A sublayer model for the deposition of particles from a turbulent flow. *Chem. Engng Sci.* **30**, 983–992.

COMTE-BELLOT, G. 1965 *Encoulement Turbulent entre Deux Parois Parallels.* Publications Scientifiques Techniques du Minestere de lair, No. 418.

COUSINS, L. B. & HEWITT, G. F. 1968 Liquid phase mass transfer in annular two-phase flow: droplet deposition and liquid entrainment. AERE-R 5657.

DAVIES, C. N. 1966 Deposition of aerosols from turbulent flow through pipes. *Proc. R. Soc.* **A289**, 235–246.

FARMER, R. A., GRIFFITH, P. & ROHSENOW, W. M. 1970 Liquid droplet deposition in two-phase flow. *Trans. ASME, J. Heat Transfer* **92**, 587–594.

FORNEY, L. J. & SPIELMAN, L. A. 1974 Deposition of coarse aerosols from turbulent flow. *J. Aerosol. Sci.* **5**, 257–271.

FRIEDLANDER, S. K. 1954 Deposition of suspended particles from turbulent gases. Ph.D. thesis, Univ. of Illinois, Urbana.

FRIEDLANDER, S. K. 1957 Behavior of suspended particles in a turbulent fluid. *AIChEJ.* **3**, 381–385.

FRIEDLANDER, S. K. & JOHNSTONE, H. F. 1957 Deposition of suspended particles from turbulent gas streams. *Indust. Engng Chem.* **49**, 1151–1156.

GARDNER, G. S. 1975 Deposition of particles from a gas flowing parallel to a surface. *Int. J. Multiphase Flow* **2**, 213–218.

HAGIWARA, Y. & SATO, T. 1979 An experimental investigation on liquid droplets diffusion in annular-mist flow. *2nd Multi-phase Flow and Heat Transfer Symp.-Workshop*, Miami, FL; also published in *Multiphase Transport* (Edited by T. N. Verziroglu). Hemisphere, New York.

HAY, J. S. & PASQUILL, F. 1957 Diffusion from a fixed source at a height of a few hundred feet in the atmosphere. *J. Fluid Mech.* **2**, 299–310.

HINZE, J. O. 1975 *Turbulence*, 2nd Edn. McGraw-Hill, New York.

HUTCHINSON, P. HEWITT, G. F. & DUKLER, A. E. 1971 Deposition of liquid or solid dispersions from turbulent gas streams; a stochastic model. *Chem. Engng Sci.* **26**, 419–439.

ILORI, T. A. 1971 Turbulent deposition of particles inside pipes. Ph.D. thesis in Mech. Engng, Univ. of Minnesota.

JAMES, P. W. & HUTCHINSON, P. 1979 Droplet deposition in an annular geometry. *Int. J. Multiphase Flow* **5**, 103–112.

KAYS, W. M. 1966 *Convective Heat and Mass Transfer.* McGraw-Hill, New York.

KOMASAWA, I. KUBOI, R. & OTAKE, T. 1974 Fluid and particle motion in turbulent dispersion—I. Measurement of turbulence of liquid by continual pursuit of tracer particle motion. *Chem. Engng Sci.* **29**, 641–650.

LAUFER, J. 1954 The structure of turbulence in fully developed pipe flow. NACA Rep. 1174.

LIU, B. Y. H. & AGARWAL, J. K. 1974 Experimental observation of aerosol deposition in turbulent flow. *J. Aerosol Sci.* **5**, 145–155.

LIU, B. Y. H. & ILORI, T. A. 1974 Aerosol deposition in turbulent pipe flow. *Environ. Sci. and Tech.* **8**, 351–356.

LUMLEY, J. L. 1978 Two-phase and non-Newtonian flows. *Turbulence* (Edited by P. Bradshaw), *Topics in Applied Physics*, Vol. 12, pp. 289–324.

MARTIN, G. Q. & JOHANSON, L. N. 1965 Turbulence characteristics of liquids in pipe flow. *AIChEJ.* **11**, 29–33.

MASTANAIAH, K. 1980 Experimental and theoretical investigation of droplet deposition and heat transfer in air–water dispersed flow. Ph.D. thesis, Energy Engng Dept., Univ. of Illinois, Chicago.

MCCOY, D. D. 1975 Particle deposition in turbulent flow. M. S. Thesis, Dept. Chem. Engng, Univ. of Illinois at Urbana-Champaign.

MCCOY, D. D. & HANRATTY, T. J. 1977 Rate of deposition of droplets in annular two phase flow. *Int. J. Multiphase Flow* **3**, 319–331.

MONIN, A. S. & YAGLOM, A. M. 1971 *Statistical Fluid Mechanics: Mechanics of Turbulence* (Edited by J. L. Lumley). MIT Press, Cambridge.

NAMIE, S. & UEDA, T. 1972 Droplet transfer in two-phase annular mist flow—I. Experiment of droplet transfer rate and distributions of droplet concentration and velocity. *Bull. J.SME* **15**, 1568–1580.

NAMIE, S. & UEDA, T. 1973 Droplet transfer in two-phase annular mist flow—II. Prediction of droplet transfer rate. *Bull. J.SME* **16**, 752–764.

NUKIYAMA, S. & TANASAWA, Y. 1938 & 1939 An experiment on the atomization of liquid by means of an air stream. Reps. 1 and 4. *Trans. J.SME* **4**, 86, *and* **5**, 68.

OWEN, P. R. 1969 Pneumatic transport. *J. Fluid Mech.* **39**, 407–432.

REEKS, M. W. & SKYRME, G. 1976 The dependence of particle deposition velocity on particle inertia in turbulent pipe flow. *J. Aerosol Sci.* **7**, 485–495.

REICHARDT, H. 1951 Vollständige Darstellung der turbulenten Geschwindigkeitsverteilung in glatten Leitungen. *ZAMM* **31**, 208–219.

ROUHIAINEN, P. O. & STACHIEWICZ, J. W. 1970 On the deposition of small particles from turbulent streams. *Trans. ASME, J. Heat Transfer* **92**, 169–177.

SCHWENDIMEN, L. C. & POSTMA, A. K. 1961 Turbulent deposition in sampling lines. USAEC Rep. HW-65309.

SEHMEL, G. A. 1968 Aerosol deposition from turbulent air streams in vertical conduits. Pacific Northwest Laboratory Rep. BNWL-578.

SEHMEL, G. A. 1970 Particle deposition from turbulent air flow. *J. Geophys. Res.* **75**, 1766–1781.

SNYDER, W. H. & LUMLEY, J. L. 1971 Some measurements of autocorrelation functions in a turbulent flow. *J. Fluid Mech.* **48**, 41–71.

SOO, S. L. 1967 *Fluid Dynamics of Multiphase Systems.* Blaisdell Press, London.

SOO, S. L. & TUNG, S. K. 1972 Deposition and entrainment in pipe flow of suspension. *Powder Tech.* **6**, 283–294.

SOO, S. L. 1978 Diffusivity of spherical particles in dilute suspensions. *AIChE J.* **74**, 184–185.

TATTERSON, D. F., DALLMAN, J. C. & HANRATTY, T. J. 1977 Drop sizes in annular gas–liquid flows. *AIChE J.* **23**, 68–76.

TCHEN, C. M. 1947 *Mean Value and Correlation Problems Connected with the Motion of Small Particles Suspended in a Turbulent Fluid.* Martinus Nijhoff, The Hague.

TENNEKES, H. & LUMLEY, J. L. 1972 *A First Course in Turbulence.* MIT Press, Cambridge.

TROUNG QUANG MINH & HUYGHE, J. 1965 Some hydrodynamical aspects of annular dispersed flow: Entrainment and film thickness. *Symp. Two-Phase Flow*, Paper C2, Exeter, England.

WELLS, A. C. & CHAMBERLAIN, A. C. 1967 Transport of small particles to vertical surfaces. *Br. J. Appl. Phys.* **18**, 1793–1799.